Int J Theor Phys (2009) 48: 1247–1262
DOI 10.1007/s10773-008-9897-5

Phantom Divide Crossing on Brane World

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Received: 8 September 2008 / Accepted: 7 November 2008 / Published online: 19 November 2008

Abstract We study the RSII brane world corrected by the four-dimensional scalar curvature and five-dimensional Gauss-Bonnet curvature. The energy transfer between brane and bulk is also taken into account. Parameterizing the energy transfer, the resulting Friedmann equation on brane is solved at low energy. It is shown that phantom divide crossing may be achieved in the braneworld model with wide possibilities, embodying the combined effect of brane-bulk energy transfer, curvature corrections, and the fine-turning mechanics.

Keywords Phantom · Brane world · Friedmann equation

1 Introduction

The recent cosmological observations leads to an important opportunity for changing the basic concept of the gravitation. A "dark energy" component with negative pressure was suggested to account for the invisible fuel that drives the current acceleration of the universe. The simplest candidate of dark energy is the cosmological constant, which gives the equation of state (EoS) of the dark energy $\omega = -1$. It does nicely well at the pragmatic observational level, but entails seemingly unsurmountable problems on the theoretical side. The dynamical dark energy are also proposed, including quintessence [15, 28, 41, 52, 57], phantom [14, 16, 25], and so on. The EoS of quintessence is always $\omega > -1$. The phantom [14, 16, 25] is the simplest model with $w < -1$ but which violates the null energy condition. Moreover, the EoS of some dark energy models can be varied from $\omega > -1$ to $\omega < -1$, which called "crossing the phantom divide".

Though many estimates are model dependent, it has been found [46–48] that the most observational probes indeed mildly favor dynamically evolving dark energy crossing the

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phantom divide at $z \sim 0.25$. Moreover, in the period of structure formation, a highly negative ω makes negligible the undesirable dark energy contribution to the total energy density. These results increases the interest on studying the possibility of phantom divide crossing. The most dark energy model with this possibility either consist of multiple components with at least one non-canonical phantom component [3, 22, 40, 65, 66] or have recourse to extend gravity theory [9, 18, 24, 26, 50, 54, 55]. The former is usually plagued by catastrophic UV instabilities or is regarded as an effective field description following an underlying theory with positive energies [17, 49], and the latter approach is severely constrained by local solar system, for example the so called $1/R$ gravity [20, 21, 33, 45, 51, 56]. Other model with phantom divide crossing including interacting holographic dark energy model [60–62], and models with interactions between dark matter and dark energy [12, 23, 27, 32, 59], and so on.

In recent years, theories of large extra dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional space-time, have received a lot of interest. There are two curvature corrections which are well motivated to the well-known RS braneworld model: a four-dimensional scalar curvature and a five dimensional Gauss-Bonnet curvature term. The first emerges because the localized matter fields on the brane, which couple to the bulk gravitons, can generate via quantum loops a localized four dimensional world volume kinetic terms for graviton [29, 30]. The second is the dominating quantum corrections to Einstein-Hilbert action for a ghost-free theory [10, 67]. Moreover, the Gauss-Bonnet action in five dimension is the most general action to produce the second-order field equation [43]. The combined effect of these curvature corrections can remove the infinite-density big bang singularity [11, 38], besides preserving the self-accelerating of the universe at late time. In certain realization of string theory, the ghost-free GB term in bulk action may naturally lead to DGP induced gravity term on the brane boundary [44].

Braneworld models admit a wider range of possibilities for dark energy, but usually have not the phantom divide crossing. The braneworld models with energy exchange between the brane and bulk has been studied in the different approaches [1, 2, 31, 63]. It has been proved that the energy exchange may lead to RS model with the late-time acceleration [4, 5, 34–36, 39, 58] and even phantom divide crossing, provided that there is an additional dark energy on brane [6, 13] or the bulk pressure may affect the brane [7, 8].

The aim of the present work is to study the brane cosmology with brane-bulk energy transfer at low energy. The model is constructed based on an extended RS(II) scenario [53] by considering the scalar curvature and GB curvature corrections. The combined effect of these corrections at very low energy has been discussed in our previous work [64]. We obtained a closed system of three equations which determines the evolvement of square of Hubble rate, the energy density on brane, and an auxiliary variable ψ_1, which takes role as the correction to bulk cosmological constant Λ, that is just the dark radiation term in RS model. Parameterizing the energy transfer by Hubble rate and scale factor, we solved the dynamic system for the resulting Friedmann equation on brane at very low energy, where one of the key condition is $\psi_1 \ll \Lambda$. However, since this model is effective at very low energy, we checked the model only using the low redshift datasets like SnIa, while not the 3-year WMAP CMB shift parameter which focuses on the high redshift region. In this paper, as a preliminary step to solve the dynamic system in the whole energy region, we will release the condition a little, using $\psi_1 < \Lambda$ instead. We will discuss the two corrections and two energy regions in detail. The main result of this paper is that the phantom divide crossing may be achieved in the brane world with wide possibilities. Moreover, an interesting result is that the Friedmann equations in energy region $\psi_1 < \Lambda$ can not recover the equations in $\psi_1 \ll \Lambda$ for some cases. This is because the fine-turning mechanics are different in two energy regions.

This paper is organized as follows: In Sect. 2, we introduce a general braneworld model with both curvature corrections and the energy exchange between the brane and bulk at low energy. In Sect. 3, we investigate the possibility of phantom divide crossing in RS model, DGP model, GB braneworld model, and DGP model with GB correction, respectively. In the last section, we give the conclusion and discussions.

2 Brane-World Model with Curvature Corrections and Brane-Bulk Energy Transfer

We consider a 3-brane imbedded in a five-dimensional bulk space-time. The RS gravition action with two curvature corrections is [37]

$$S_{grav} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\{L_{EH} + \alpha L_{GB}\} + \frac{1}{2\kappa_4^2}\int_{brane} d^4x\sqrt{-\tilde{g}}L_{IG}, \tag{1}$$

including five-dimensional part characterized by five-dimensional gravitational coupling constant κ_5 and four-dimensional part characterized by four-dimensional gravitational coupling constant κ_4. The $L_{EH} = R - 2\Lambda$ is the five-dimensional Einstein-Hilbert Lagrangian with cosmological constant Λ. The GB curvature correction term

$$L_{GB} = R^2 - 4R_{ABb}R^{AB} + R_{ABCD}R^{ABCD}$$

is characterized by the GB coupling α defined through string energy scale g_s, $\alpha = \frac{1}{8g_s^2}$. The induced-gravity Lagrangian $L_{IG} = \tilde{R} - 2\kappa_4^2\lambda$ that characterized by crossover length scale $r = \frac{\kappa_5^2}{\kappa_4^2}$, consists of four-dimensional scale curvature $\tilde{R}$ and brane tensor λ. The pure GB correction and pure induced gravity correction correspond to the case with $r = 0$ and $\alpha = 0$, respectively. And it is RS model with $r = \alpha = 0$.

Varying (1) with respect to the bulk metric, the field equation can be written as

$$G_{AB} + 2\alpha H_{AB} = \kappa_5^2 T_{AB}|_{total}, \tag{2}$$

where H_{AB} is second order Lovelock tensor [43]

$$H_{AB} = RR_{AB} - 2R_A^C R_{BC} - 2R^{CD}R_{ABCD} + R_A^{CDE}R_{BCDE} - \frac{1}{4}g_{AB}L_{GB}.$$

The total energy-momentum tensor $T_{AB}|_{total}$ include bulk and brane components

$$T_{AB}|_{total} = T_{AB}|_{bulk} + T_{AB}|_{brane}\hat{\delta}(y),$$

where we have assumed the brane is fixed at $y = 0$, and used the normalized Dirac delta function, $\hat{\delta}(y) = \sqrt{\tilde{g}/g}\,\delta(y)$. The bulk component is

$$T_{AB}|_{bulk} = -\Lambda g_{AB} + T_{AB},$$

where T_{AB} is any possible energy-momentum in the bulk, and the brane component is

$$T_{AB}|_{brane} = -\lambda\tilde{g}_{AB} - \frac{r}{\kappa_5^2}\tilde{G}_{AB} + \tilde{T}_{AB},$$

where the second term is the contributions arising from the scalar curvature and $\tilde{T}_{AB}$ is taken to be the tensor of perfect fluid with energy density ρ and constant EoS w_m. Hereafter, for convenience, we will absorb the κ_5^2 in energy-momentum tensor.

For homogeneous and isotropic cosmology, we consider the 5-dimensional FRW line element

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{3}$$

where γ_{ij} is a constant curvature three-metric, with curvature index $k = 0, \pm 1$ and $n(t,0) = 1$, so that t is proper time along the brane. We are interest in spatially flat brane $k = 0$ and assume the fifth dimension is static $\dot{b} = 0$ and set $b = 1$ for convenience.

From the 00 and ij components of field equation (2), the discontinuity of the first derivative on the brane can be obtained

$$2\left[1 + \frac{8}{3}\alpha\left(H^2 + \frac{\Phi}{2}\right)\right]\frac{a'}{a} = rH^2 - u, \tag{4}$$

$$2\frac{n'}{n} + 4\frac{a'}{a} + 8\alpha\frac{n'}{n}\Phi + 16\alpha\frac{a'}{a}(H^2 + \dot{H}) = 3r\left(H^2 + \frac{2}{3}\dot{H}\right) + v, \tag{5}$$

where a prime stands for the derivative with respect to y, a dot denotes the derivative with respect to t,

$$\Phi = \frac{H^2}{n^2} - \frac{a'^2}{a^2}$$

and

$$u = \frac{1}{3}(\rho + \lambda), \qquad v = (w_m\rho - \lambda).$$

It has been found that (4) implies an algebra cubic equation about the square of Hubble rate H^2

$$4\left[1 + \frac{8}{3}\alpha\left(H^2 + \frac{\Phi}{2}\right)\right]^2 (H^2 - \Phi) = [rH^2 - u]^2 \tag{6}$$

and its solution will give the desired Friedmann equation, if we know the undetermined variables Φ and ρ.

Consider ρ first. As a consequence of the divergence-free character of the left-hand side of (2), we have Bianchi identity $\nabla_A T_B^A|_{total} = 0$. Integrate its zero component around $y = 0$ and using the Z_2 symmetry, we can determine the evolution of ρ

$$\dot{\rho} + 3(1 + w_m)H\rho = 2T_{05}. \tag{7}$$

This denotes that the energy conservation law on brane is broken. The discontinuity of the first derivative from cubic equation (4) are [64]

$$\frac{a'}{a} = \frac{1}{2}\left(rH^2 - u\right) + \frac{1}{6}\left(rH^2 - u\right)\left[r^2H^4 + u^2 - 2H^2(6 + ru)\right]\alpha, \tag{8}$$

$$\frac{n'}{n} = \frac{1}{2}\left(2u + v + rH^2 + 2r\dot{H}\right) + \frac{1}{6}\{r^3H^6 + 3rH^4(-4 + 2ru + rv + 2r^2\dot{H}) + u[u(8u + 3v) + 6(4 + ru)\dot{H}] - 3H^2[4v + u(8 + 5ru + 2rv) + 4r(4 + ru)\dot{H}]\}\alpha. \tag{9}$$

Substituting (8) and (9) into 05 component of field equation (2), we recover (up to first α order) the same semi-conservation law equation (7).

The above first integrals can be recovered from a general differential equation with non-trivial T_{05} and T_{55}

$$\dot{\psi} + 4H\psi + 4T_{05}\frac{a'}{a} + 4H(T_{55} - \Lambda) = 0, \tag{10}$$

where

$$\psi = 6(\Phi + 2\alpha\Phi^2).$$

The physical meaning of ψ can be understood as the effective bulk cosmological "constant". This may be more obvious to change (10) as

$$\dot{\psi}_1 + 4H\psi_1 + 4T_{05}\frac{a'}{a} + 4HT_{55} = 0, \tag{11}$$

where $\psi_1 = \psi - \Lambda$ takes role as the correction to bulk cosmological constant. For $T_{05} = T_{55} = 0$, the modification for usual bulk cosmological constant Λ comes from the dark radiation term $\sim a^{-4}$. For a nontrivial bulk, the term $4T_{05}\frac{a'}{a} + 4HT_{55}$ in (11) will further correct Λ and the dark radiation.

Generally, (6) has three solutions of H^2, but only two of them are left for small α:

$$\begin{aligned} H^2 = {} & \frac{6 + 3ru \mp \phi}{3r^2} + \frac{\alpha}{9r^4\phi}\{32[\mp 9r^2u + 6ru\phi \mp 2\phi(\mp 6 + \phi)] \\ & \pm 8r^2(\pm 18 + 3ru + \phi)\psi \pm r^4\psi^2\}, \end{aligned} \tag{12}$$

where $\phi = \sqrt{6}\sqrt{6 + 6ru - r^2\psi}$. These two branches recover the two branches of DGP model when $\alpha \to 0$, see the first term. For GB brane world only the up branch has right limit under $r \to 0$,

$$H^2 = \frac{u^2}{4} + \frac{\psi}{6} - \frac{\alpha}{18}(6u^4 + 6u^2\psi + \psi^2). \tag{13}$$

One can find three equations (7), (11) and (12) consist of a closed system for variables ρ, ψ (or ψ_1) and H, which determines the evolution of universe provided we have known bulk energy-momentum tensor. Unfortunately, it is not yet available and obviously depends on mechanism which produces the energy transfer. We consider the ansatz $T_{05} = THa^n$ (T is a constant) which is taken in [6–8, 13] for RS brane world. Then one can find the total energy density

$$\rho = \rho_0 a^{-3} + \frac{2T}{3+n}a^n, \tag{14}$$

where we take $w_m = 0$ for dark matter and ρ_0 is an integral constant. If we introduce a non-phantom dark energy with EoS w on brane as Ref. [6, 13], and assume the dark matter is conserved, then the total energy density should be changed as

$$\rho = \rho_1 a^{-3} + \rho_2 a^{-3(1+w)} + \frac{2T}{3+3w+n}a^n, \tag{15}$$

where ρ_1, ρ_2 are integral constants and $-1 < w < 0$ for preserving the null energy condition. Observing (7) and (14) or (15), we notice an order in general

$$Ta^n \lesssim \rho. \tag{16}$$

Equation (11) determining the correction to bulk cosmological constant is very complicated in general. In the rest of paper, we will seek the effective solution at low energy $\rho \ll \lambda$. Moreover, to derive a cosmological system that is largely independent of the bulk dynamics, usually one can assume on the brane the contribution of T_{55} relative to the bulk vacuum energy is much less important than the brane matter relative to the brane vacuum energy, or schematically $\frac{T_{55}}{\Lambda} \ll \frac{\rho}{\lambda}$, which leads that T_{55} can be omitted at low energy $\rho \ll \lambda$.

3 RS Model

For the sake of comparison, let us discuss the model under $\alpha \to 0$ and $r \to 0$ limit. From the equation of Hubble rate (12), we have

$$H^2 = A_1\rho + A_2\psi_1 + A_3, \tag{17}$$

where

$$A_1 = \frac{\lambda}{18}, \qquad A_2 = \frac{1}{6}, \qquad A_3 = \frac{1}{36}(\lambda^2 + 6\Lambda), \tag{18}$$

and the higher order term ρ^2 has been omitted at low energy $\rho \ll \lambda$. From the junction condition (8), we find

$$\frac{a'}{a} = B_1,$$

where

$$B_1 = -\frac{1}{6}\lambda. \tag{19}$$

Then (11) can be simplified as

$$\dot{\psi}_1 + 4H\psi_1 + 4T_{05}B_1 = 0. \tag{20}$$

It can be directly integrated

$$\psi_1 = Ca^{-4} - \frac{4B_1}{4+n}Ta^n, \tag{21}$$

where C is an integral constant. Substituting ψ_1 (21) and ρ (15) into (17), one can obtain the Friedmann equation on brane

$$H^2 = \Omega_{0m}a^{-3} + \Omega_{0w}a^{-3(1+w)} + \Omega_{0d}a^{-4} + \Omega_{0n}a^n, \tag{22}$$

where

$$\begin{aligned} \Omega_{0m} &= A_1\rho_1, \qquad \Omega_{0w} = A_1\rho_2, \qquad \Omega_{0d} = A_2 C, \\ \Omega_{0n} &= T\left(\frac{2A_1}{3+3w+n} - \frac{4A_2B_1}{4+n}\right). \end{aligned} \tag{23}$$

It should be emphasized that the effective cosmological constant on brane A_3, which is composed by constant terms λ^2 and Λ, has been tuned to zero using well known RS relation

$\Lambda = -\frac{1}{6}\lambda^2$, otherwise since $A_1\rho \ll A_3$, the linear ρ term in Friedmann equation may be neglected, which is conflicted with observation. Neglecting the dark radiation term, it recovers the Friedmann equation obtained in Ref. [6, 13]

$$H^2 = \Omega_{0m} a^{-3} + \Omega_{0w} a^{-3(1+w)} + \Omega_{0n} a^n. \tag{24}$$

Using $a = \frac{a_0}{1+z}$, the EoS of effective dark energy can be defined as [19, 42]

$$w_{eff} = -1 + \frac{1}{3}\frac{d\log[H^2 - \Omega_{0m}(1+z)^3]}{d\log(1+z)},$$

where we have absorbed a_0 into parameters. Assume $w_{eff} = -1$ at z_T, then the EoS at present

$$w_{eff}|_{z=0} = -1 + \frac{n(1+w)[1-(1+z_T)^{(1+w)+n}]}{n + 3(1+w)(1+z_T)^{(1+w)+n}}, \tag{25}$$

where we have used unit $H^2|_{z=0} = 1$. In Ref. [6, 13], using the prior of $z_T = 0.2$, as indicated by extensive analysis of observational data, and fitting the recent type Ia supernova, SDSS and WMAP data, it is shown that the modified gravity on the brane with phantom divide crossing is consistent with observations.

We will give several points as comment to this model. First, observing (25), we find that under $z_T > 0$ and $-1 < w < 0$, EoS $w_{eff}|_{z=0} < -1$ needs $n > 0$, which implies that the brane-bulk energy flow increases along expansion of universe (respecting $H \sim H_0$ at late time). In the rest of this paper, we will show that for the case $n = 0$, which implies when the brane-bulk energy flow is invariable along expansion of universe, and the case $n < 0$, which implies that the brane-bulk energy flow decreases along expansion of universe, are all permitted to cross the phantom divide in the model with curvature corrections. Second, usually the dark radiation term is neglected if one considers the large scale factor at late time. However, it may be held since one indeed do not know the constant C which reflects the bulk geometry (Notice that it takes role as the bulk black hole mass in a Schwarzschild-AdS_5 geometry). Now we hold the dark radiation term, but consider ρ as (14) which dose not introduce the additional dark energy on brane. Similar to obtain (22), we have

$$H^2 = \Omega_{0m} a^{-3} + \Omega_{0d} a^{-4} + \Omega_{0n} a^n, \tag{26}$$

where Ω_{0n} should be replaced with

$$\Omega_{0n} = T\left(\frac{2A_1}{3+n} - \frac{4A_2B_1}{4+n}\right).$$

The present EoS

$$w_{eff}|_{z=0} = -1 + \frac{4n[1-(1+z_T)^{4+n}]}{3[n+4(1+z_T)^{4+n}]}.$$

We find that the phantom divide crossing may also be achieved under $n > 0$. It results an attractive feature that the nontrivial low energy cosmology is caused merely by geometric effect. For an explicit example of the phantom divide crossing and current accelerated expansion, which is characterized by deceleration factor $q = -\frac{\ddot{a}/a}{H^2}$

$$q = -1 - \frac{1}{2}\frac{d\log H^2}{d\log(1+z)}, \tag{27}$$

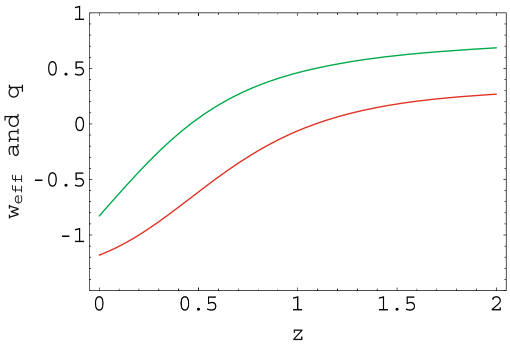

Fig. 1 (Color online) GermaniThe EoS w_{eff} (*red line*) with prior $z_T = 0.2$, $n = 1$ and deceleration factor q (*green line*) with prior $z_T = 0.2$, $n = 1$, $\Omega_{0m} = 0.25$ versus redshift z. One can find that w_{eff} crosses -1, and the q crosses 0 at $z \sim 0.5$ and the order of magnitude $q|_{z=0} \sim -1$

see Fig. 1.

Third, observing the Friedmann equation in RS model (17), one can find that the low energy condition $\rho \ll \lambda$ and RS fine-turning mechanics imply the order

$$\rho \ll \lambda, \qquad \Lambda \sim \lambda^2, \qquad H^2 \ll \lambda^2, \qquad \psi_1 \ll \Lambda \tag{28}$$

Compared with $\rho \ll \lambda$ on brane, the last inequality can be understood as a dual of brane and bulk. In the rest of paper, we will use it to find Friedmann equations in more complicated models.

4 DGP Brane World

In this section, we will study the DGP brane world. The desired Friedmann equation on brane is determined by (11), (12) and (7) under limit $\alpha \to 0$.

4.1 A Low-Energy Effective Theory

We will first consider a simple low-energy effective theory. We assume $\psi_1 \ll \Lambda$. Respecting $\rho \ll \lambda$, (12) can be expanded versus ρ and ψ_1. Up to first order, we obtain the same form as (17)

$$H^2 = A_1\rho + A_2\psi_1 + A_3, \tag{29}$$

where only the constants are different

$$A_1 = \frac{6 + r\lambda \mp D}{3r^2}, \qquad A_2 = \frac{D \mp 6}{3Dr}, \qquad A_3 = \pm\frac{1}{D}, \tag{30}$$

where $D = \sqrt{6(6 + 2r\lambda - r^2\Lambda)}$. One can find that there are two branches for DGP model. For simplicity, we will consider only above branch hereafter. Similarly, (8) can be expanded

$$\frac{a'}{a} = B_1 + B_2\rho + B_3\psi_1, \tag{31}$$

where

$$B_1 = \frac{6 - D}{6r}, \qquad B_2 = -\frac{1}{D}, \qquad B_3 = \frac{r}{2D}. \tag{32}$$

One can find the order $\frac{B_1}{B_2} \gtrsim \lambda$ and $\frac{B_1}{B_3} \gtrsim \Lambda$ in general. By considering $\rho \ll \lambda$ and $\psi_1 \ll \Lambda$, the later two terms in (31) can be omitted, so

$$\frac{a'}{a} = B_1. \tag{33}$$

Correspondingly, (20) is recovered with different constant B_1

$$\dot{\psi}_1 + 4H\psi_1 + 4T_{05}B_1 = 0.$$

Thus, we can find that the scale curvature correction in three equations (7), (11) and (12) only affects the constants A_1, A_2, A_3, B_1 finally, and they will not vanish under $r \to 0$ limit. As a consequence, the curvature correction will not change the form of Friedmann equation, and the corresponding evolution of universe which predicted by fitting the parameters in Friedmann equation to observation datasets. This result is independent with the concrete mechanics to produce the brane-bulk energy exchange T_{05}.

However, it should be noticed that the change of parameters may play nontrivial role in some cases. For example, considering $T_{05} = THa^n$ and using (14), Friedmann equation (26) can be recovered with the parameters A_1, A_2, A_3, B_1 replaced by (30) and (32). To impose the vanishing effective cosmological constant on brane, we choice $A_3 = 0$. One can find that it just recovers RS fine-tuning mechanics. It has been noticed in above section that, in RS model, the reason that we can hold the dark radiation term is that we do not know the constant C which reflects the bulk geometry. In current model, furthermore, the curvature corrections embodied in constants (30) and (32) may strength the possibility to hold the dark radiation term. For an explicit case, we assume $C \ll 6\rho_1\lambda a$. Noticing (18), one can find that the dark radiation term can be omitted since $\Omega_{0d}a^{-4} \ll \Omega_{0m}a^{-3}$ in RS case. For holding the dark radiation term i.e. $\Omega_{0d}a^{-4} \sim \Omega_{0m}a^{-3}$ in DGP model, we need $C \sim \frac{A_1\rho_1 a}{A_2} \ll 6\rho_1\lambda a$ i.e. $\frac{A_1}{A_2} \ll 6\lambda$. Using (30), one can find the condition is satisfied when

$$r \gg \frac{38\lambda}{54\lambda^2 + \Lambda}. \tag{34}$$

Another example can be given by considering the relationship between the accelerated expansion and brane-bulk energy flow. We assume that the dark radiation is neglected and consider only one dark component on brane, which leads the density is determined by (14). From (27), one can find that the present accelerated expansion $q|_{z=0} < 0$ needs $n > 0$ and $\Omega_{0n} > 0$ which suggests in RS case (noticing $\lambda > 0$ in RS(II) brane world), the bulk energy must flow into brane (the later term in (14) is positive, i.e. $T > 0$). However, if we consider the curvature correction, one can find that accelerated expansion also may be achieved when the bulk energy flows out brane, which only needs

$$\left|\frac{A_1}{A_2}\right| < \frac{n+3}{3(n+4)}\lambda, \tag{35}$$

that can be easily realized.

At last, we give a possible low energy region to realize $\psi_1 \ll \Lambda$. Let us consider the order of some quantities. We care about ρ and H^2 at the low energy region

$$\rho \ll \lambda, \qquad \rho \ll \frac{\Lambda}{\lambda}, \qquad H^2 \ll \frac{\lambda}{r}. \tag{36}$$

We points out that this low energy region is self-consistent and can be easily realized. The most simple case is to assume that the brane tensor and bulk cosmological constant are very big. Also, one should be noticed that the low energy region under $r \to 0$ limit is consistent with RS low energy region (28), and if one respects $rH \sim 1$ in DGP(+) brane, $rH^2 \ll \lambda$ is not a new constrain beyond RS low energy region (28). Considering (16), (33) and (36), we have $T_{05}\frac{a'}{a} \ll H\Lambda$. Recalling the discussion about the correction to bulk cosmological constant below (11), now one can conclude that the correction to bulk cosmological constant ψ_1 is very small than bulk cosmological constant, or schematically, $\psi_1 \ll \Lambda$. Moreover, one can find the conditions (34), (35) for two nontrivial effect of curvature correction do not violate the low energy region (36).

4.2 A More General Case

Now we will consider more general case $\psi_1 < \Lambda$. Equation (12) still can be expanded as (29) and (8) can be expanded as (31), however here only the second term can be omitted

$$\frac{a'}{a} = B_1 + B_3\psi_1. \tag{37}$$

Correspondingly, (11) can be simplified as

$$\dot{\psi}_1 + 4H\psi_1 + 4T_{05}(B_1 + B_3\psi_1) = 0. \tag{38}$$

Integrating this equation, we have

$$\psi_1 = \frac{-B_1 T}{1 + B_3 T} + C_1 a^{-4-4B_3 T}, \quad n = 0, \tag{39}$$

$$\psi_1 = -\frac{B_1}{B_3} + C_2 a^{-4} e^{-\frac{4B_4}{n}Ta^n} - \frac{4B_1}{nB_3} e^{-\frac{4B_4}{n}Ta^n} E_{\frac{n-4}{n}}\left(-\frac{4Ta^n B_4}{n}\right), \quad n \neq 0 \tag{40}$$

where the exponential integral function satisfies $E_m(x) = \int_1^\infty t^{-m} e^{-xt} dt$, and C_1, C_2 are two integral constants. Substitute (40) into (29),

$$H^2 = \left(A_3 - A_2\frac{B_1}{B_3}\right) + A_1\rho + A_2\left[C_2 a^{-4} e^{-\frac{4B_4}{n}Ta^n} - \frac{4B_1}{nB_3} e^{-\frac{4B_4}{n}Ta^n} E_{\frac{n-4}{n}}\left(-\frac{4Ta^n B_4}{n}\right)\right].$$

Notice that there is no constant in (15) when $n \neq 0$ hence the constant in H^2 is $A_3 - A_2\frac{B_1}{B_3} = \frac{\lambda}{3r}$. Comparing it with $A_2\rho$, we find $A_2\rho \ll \frac{\lambda}{3r}$, which makes the desired linear ρ term may be omitted in H^2. Thus this solution is not reasonable. The left solution can be obtained by substituting (39) and (15) into (29). We recover (24), but the parameters Ω_{0n} and n in (23) are changed

$$\Omega_{0n} = A_2 C_1, \quad n = -4(1 + B_3 T).$$

We note that the cosmological constant on brane $A_3 - \frac{A_2 B_1 T}{1+B_3 T}$ has been turned into zero. Obviously, the phantom divide crossing can be achieved if $n > 0$ i.e. $B_3 T < -1$. We conclude that the effect of induced gravity is necessary for the phantom divide crossing.

5 GB Brane World

In this section, we study the GB brane world. Considering $\rho \ll \lambda$, the Friedmann equation (13) can be expanded as

$$H^2 = A_1\rho + A_2\psi_1 + A_3 + A_4\psi_1^2, \tag{41}$$

where

$$\begin{aligned}
A_1 &= -\frac{1}{486}\lambda\left(-27 + 8\alpha\lambda^2 + 36\alpha\Lambda\right),\\
A_2 &= \frac{1}{54}\left[9 - 2\alpha\left(\lambda^2 + 3\Lambda\right)\right],\\
A_3 &= \frac{1}{972}\left[-4\alpha\lambda^4 - 54\Lambda\left(-3 + \alpha\Lambda\right) - 9\lambda^2\left(-3 + 4\alpha\Lambda\right)\right],\\
A_4 &= -\frac{\alpha}{18}.
\end{aligned}$$

Substituting it into junction condition (8) under $r \to 0$ limit, (11) can be simplified at low energy, which recovers (38)

$$\dot{\psi}_1 + 4H\psi_1 + 4T_{05}(B_1 + B_3\psi_1) = 0, \tag{42}$$

where

$$B_1 = -\frac{\lambda}{6} + \frac{\alpha\lambda^3}{81} + \frac{\alpha\lambda\Lambda}{9}, \qquad B_3 = \frac{\alpha\lambda}{9}.$$

If we consider the case $\psi_1 \ll \Lambda$, the $A_4\psi_1^2$ in (41) and $B_3\psi_1$ in (42) can be omitted in general. Obviously, using (15), similar to the discussion of previous sections, we can recover the Friedmann equation (22) except a small correction to constants (23) concerning the small GB coupling. We will not repeat the derivation, but give a possible low energy region to realize $\psi_1 \ll \Lambda$ in GB model

$$\rho \ll \lambda, \qquad \rho \ll \frac{\Lambda}{\lambda}, \qquad H^2 \ll \lambda^2. \tag{43}$$

To preserve the efficiency of the expansion (8), we notice $\alpha\lambda^2 < 1$ which implies $\rho \ll \frac{\Lambda}{\alpha\lambda^3}$ under low energy region (43). Thus, similar to the discussion below (36), one can obtain $\psi_1 \ll \Lambda$.

Now we will discuss the exact Friedmann equation in GB brane world even without using $\psi_1 < \Lambda$. The solution of (42) has the same form of (39) and (40) in DGP model. However, there is an important difference with DGP model. To preserve the efficiency of expansion (13), noticing that we need $\alpha\psi < 1$, (which means $\alpha\psi_1 < 1$ and $\alpha\Lambda < 1$ in general, one can find that unless the integral constant C_2 in (40) is related to GB coupling α, the solution is not reasonable except for $-4 < n \leq 0$. For explicit, we give the typical cases $n = -1$ and $n = 1$. We will consider the expansion by B_3 for convenience, which is equivalent to expansion by α

$$\psi_1 = \frac{-4B_1T}{3a} + \frac{8B_1B_3}{3}\left(Ta^{-1}\right)^2 + \frac{C_1}{a^4} + \frac{4B_3TC_1}{3a^5}, \quad n = -1, \tag{44}$$

$$\psi_1 = \frac{-3B_1}{32\,(Ta)^4\,B_3^5} + \frac{3B_1}{8\,(Ta)^3\,B_3^4} - \frac{3B_1}{4\,(Ta)^2\,B_3^3} + \frac{B_1}{(Ta)\,B_3^2} - \frac{B_1}{B_3} + \frac{C_2}{a^4} - \frac{4TB_3C_2}{a^3}, \quad n = 1. \tag{45}$$

For $n = 1$, respecting (16) and $\alpha\lambda^2 < 1$, ψ_1 (45) is dominated by the first term which will violate the constrain $\alpha\psi_1 < 1$ at low energy

$$\alpha\psi_1 \sim \frac{\alpha B_1}{(Ta)^4 B_3^5} > \frac{1}{\rho^4\lambda^4\alpha^4} > \frac{\lambda^4}{\rho^4} \gg 1.$$

We will study whether or not the Friedmann equation of the typical case $n = -1$ may achieve phantom divide crossing. Respecting (16) and $\alpha\lambda^2 < 1$, one can find the second term and the last term of r.h.s in (44) can be omitted at low energy $\rho \ll \lambda$

$$\psi_1 = \frac{-4B_1T}{3a} + \frac{C_1}{a^4}. \tag{46}$$

Since there is not linear ρ term in ψ_1, we have the order $A_2\psi_1 \lesssim A_1\rho$ so that the linear ρ term in (41) can be held. Considering further $\alpha\lambda^2 < 1$ and $\alpha\Lambda < 1$, one can obtain $\psi_1 \lesssim \lambda\rho$. Thus, $A_4\psi_1^2$ in (41) can be omitted

$$A_4\psi_1^2 \lesssim \alpha\lambda^2\rho^2 \ll \alpha\lambda^3\rho \lesssim A_1\rho.$$

Substituting (46) into (41), we can recover the Friedmann equation (22) with $n = -1$. Notice that the effective cosmological constant on brane A_3 has been tuned to zero. Omitting dark radiation term, we recover (24) with $n = -1$. As discussion in Sect. 3, the EoS can not cross the phantom divide under $z_T > 0$ and $-1 < w < 0$. The other cases for $-4 < n \le 0$ are similar. Thus, it seems that the universe has not phantom divide crossing or rejects the GB correction. It is not expected. However, if the integral constant C_2 is related to GB coupling α, the situation will be different. Assume that C_2 is replaced as $C_2 \to C_3 + f(\alpha)$. To satisfy $\alpha\psi_1 < 1$, we find it can be achieved by making the ψ_1 finite under limit $\alpha \to 0$. Then we find

$$f(B_3) = -\frac{(-4TB_3)^{-4/n}\Gamma(\frac{4}{n})}{B_3 n}\left[(-4B_3T)^{4/n}B_3nC_3 - 4B_1n^{4/n}\right],$$

where the gamma function satisfies $\Gamma[x] = \int_0^\infty t^{x-1}e^{-t}dt$. Expanding the ψ_1 (45) up to first order, we obtain

$$\psi_1 = \frac{C_3}{a^4} - \frac{4B_1(Ta^n)}{4+n} + 4B_3Ta^n\left(\frac{2B_1(Ta^n)}{(4+n)(2+n)} - \frac{C_3}{na^4}\right).$$

Similar to simplification for (46), the ψ_1 can be simplified at low energy and at late time

$$\psi_1 = -\frac{4B_1(Ta^n)}{4+n}.$$

Substituting it into (13), we recover the Friedmann equation (24)

$$H^2 = \Omega_{0m}a^{-3} + \Omega_{0w}a^{-3(1+w)} + \Omega_{0n}a^n.$$

Obviously, the GB curvature correction only changes the parameters of the Friedmann equation (24), which has same dynamics as RS model, with desired phantom divide crossing when $n > 0$.

6 DGP Brane World with GB Correction

Now we will consider DGP brane world with GB correction. Obviously, if we assume $\psi_1 \ll \Lambda$ and use (15), we will recover the Friedmann equation (22) except the correction to constants. A possible low energy region to realize $\psi_1 \ll \Lambda$ is the combination of (36) and (43). Now we will care about the case $\psi_1 < \Lambda$. We can recover (31)

$$\frac{a'}{a} = B_1 + B_2\rho + B_3\psi_1, \tag{47}$$

where

$$\begin{aligned}
B_1 &= \frac{6-D}{6r} + \frac{\alpha}{18Dr^3}\big[192(-6+D) + 16(-18+D)r\lambda + 4(18+D)r^2\Lambda - 3r^4\Lambda^2\big],\\
B_2 &= -\frac{1}{D} + \frac{\alpha}{3D^3r^2}\big[96(-6+D) + 32(-9+D)r\lambda - 8(-27+2D)r^2\Lambda + 3r^4\Lambda^2\big],\\
B_3 &= \frac{r}{2D} + \frac{\alpha}{6D^2r}\big[-288 + 16D(3+r\lambda) - 8r^2(18+D+3r\lambda)\Lambda + 9r^4\Lambda^2\big].
\end{aligned} \tag{48}$$

One can find that $\frac{B_1}{B_2} \succeq \lambda$ in general. Hence the second term in (47) can be omitted and (37) is recovered with different constants (49). Correspondingly, (39) and (40) are recovered

$$\psi_1 = \frac{-B_1 T}{1 + B_3 T} + C_1 a^{-4-4B_3T}, \quad n = 0, \tag{49}$$

$$\psi_1 = -\frac{B_1}{B_3} + C_2 a^{-4} e^{-\frac{4B_4}{n}Ta^n} - \frac{4B_1}{nB_3} e^{-\frac{4B_4}{n}Ta^n} E_{4/n}\left(-\frac{4Ta^nB_4}{n}\right), \quad n \neq 0 \tag{50}$$

and (29) is also recovered

$$H^2 = A_1\rho + A_2\psi_1 + A_3, \tag{51}$$

where

$$\begin{aligned}
A_1 &= \frac{6+r\lambda-D}{3r^2} + \frac{\alpha}{9Dr^4}\big[-64(-6+D) + 64Dr\lambda - 32r^2\lambda^2\\
&\quad + 8r^2(-18-D+r\lambda)\Lambda + r^4\Lambda^2\big],
\end{aligned} \tag{52}$$

$$A_2 = \frac{D-6}{3Dr} + \frac{2\alpha}{9D^3r^3}\big\{32(-6+D)D^2 + 9r[-32\lambda(4+r\lambda) + 8r(8+3r\lambda)\Lambda - 3r^3\Lambda^2]\big\}, \tag{53}$$

$$A_3 = \frac{1}{D} + \frac{\alpha}{3D^3r^2}\big[-96(-6+D) - 32D(-9+D)r\lambda + 8D(-27+2D)r^2\lambda - 3r^4\Lambda^2\big]. \tag{54}$$

Substitute (50) into (51),

$$H^2 = \left(A_3 - A_2\frac{B_1}{B_3}\right) + A_1\rho + A_2\left[C_2a^{-4}e^{-\frac{4B_4}{n}Ta^n} - \frac{4B_1}{nB_3}e^{-\frac{4B_4}{n}Ta^n}E_{\frac{n-4}{n}}\left(-\frac{4Ta^nB_4}{n}\right)\right].$$

An important difference with DGP model is that the GB effect results that the effective cosmological constant on brane $A_1 - A_3\frac{B_1}{B_3}$ may be turned to zero, hence the solution $n \neq 0$

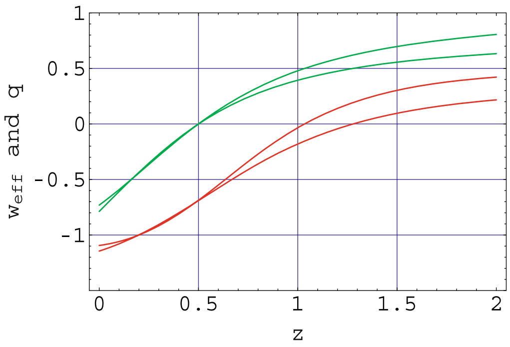

Fig. 2 (Color online) With prior $w_{eff}(0.2) = -1$, $q(0.5) = 0$, $\Omega_{0m} = 0.25$, $\Omega_{0p} = 0.1$, the EoS w_{eff} (*red lines*) and deceleration factor q (*green lines*) are plotted versus redshift z for $n = 1$ and $n = -1$ respectively. One can find w_{eff} can crosses -1, and the order of magnitude $q|_{z=0} \sim -1$. The $n = -1$ describes smaller acceleration at small redshift and higher deceleration at large redshift

is reasonable. We consider (14), then

$$H^2 = \Omega_{0m} a^{-3} + \Omega_{0n} a^n + \Omega_{0o} a^{-4} e^{\Omega_{0p} a^n} + \Omega_{0q} e^{\Omega_{0p} a^n} E_{\frac{n-4}{n}}(\Omega_{0p} a^n), \tag{55}$$

where

$$\begin{aligned} \Omega_{0m} &= A_1 \rho_1, \qquad \Omega_{0n} = \frac{2A_1 T}{3+n}, \\ \Omega_{0o} &= A_2 C_2, \qquad \Omega_{0p} = \frac{-4B_3 T}{n}, \qquad \Omega_{0q} = \frac{-4A_2 B_1}{n B_3}. \end{aligned} \tag{56}$$

One can find that the EoS of Friedmann equation (55) can cross phantom divide even when $n < 0$. For explicit, see Fig. 2.

7 Conclusion

We have studied the extended RS scenario with the brane-bulk energy transfer and two curvature corrections in detail. We consider the cosmological dynamics at low energy $\psi_1 \ll \Lambda$ and $\psi_1 < \Lambda$ respectively. We have solved the dynamic system for the Friedmann equations and study the phantom divide crossing of EoS of effective dark energy on brane. For the sake of comparison, we show that in RS model, the phantom divide crossing suggests that the brane-bulk energy flow increases along expansion of universe $n > 0$. We also show that the dark radiation term, which is neglected usually [6, 13, 54, 55], can replace the dark energy on brane introduced by hand for crossing phantom divide, consisted with recent dark energy probes. We notice that the RS model implies $\psi_1 \ll \Lambda$, which is reminiscent of $\rho \ll \lambda$ on brane and can be understood as a dual of brane and bulk. Assuming $\psi_1 \ll \Lambda$ in the model with curvature correction, which can be realized in a specific low energy region, we recover the same Friedmann equation of RS model, where the curvature corrections only change the parameters. However, we show that the change of parameters may strength the possibility to hold the dark radiation term, and the accelerated expansion also may be achieved when the bulk energy flows out brane, which is not permitted in RS model. We further discuss the more general case of $\psi_1 < \Lambda$. For the scale curvature correction, we find that the Friedmann equation is reasonable only when the brane-bulk energy flow is parameterized by Hubble rate while is invariable along expansion of universe i.e. $n = 0$. The Friedmann equation still has same dynamics of RS model, but the induced gravity is necessary for phantom divide crossing. For GB correction, we find that the phantom divide crossing acquires the integral constant of ψ_1 related to Gauss-Bonnet coupling. For combination of two curvature corrections, we find a new Friedmann equation. It is shown that the phantom divide crossing may be achieved even if the brane-bulk energy flow decreases along expansion of universe $n < 0$.

In summary, the phantom divide crossing may be achieved in brane world model with wide possibilities, embodying the combined effect of brane-bulk energy transfer and curvature corrections, without introducing phantom material violating the null energy condition.

At last, we would like to notice that the resulting Friedmann equations in energy region $\psi_1 < \Lambda$ can not always recover the equations in $\psi_1 \ll \Lambda$. To be concrete, (55) is obvious different from (26). In the model with GB curvature, the Friedmann equation without using $\psi_1 \ll \Lambda$ is reasonable only for $-4 < n \leq 0$ if the integral constant C_2 in (40) is not related to GB coupling α. However the Friedmann equation with $\psi_1 \ll \Lambda$ has not any restriction on n. We point out that the difference between the Friedmann equations is due to the different fine-turning mechanics in two energy regions. This property should be taken into account in studying the essence of fine-turning mechanics.

Acknowledgements This work was supported by the NSFC under Grant Nos. 10575068 and 10604024, the CAS Knowledge Innovation Project Nos. KJcx.syw.N2, the Shanghai Research Foundation No. 07dz22020, the Shanghai Education Development Foundation, and the Innovation Foundation of Shanghai University.

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